ChemBE 409 Homework #3 Due: 9/22/2010

1. Pharmacokinetic model formulation
   a. Sketch a process flow diagram for a pharmacokinetic model which includes a one-compartment pancreas and a two-compartment brain, connected by the bloodstream.
   b. Formulate model equations for the concentrations of a molecule in the brain. Assume the flux between the two compartments is membrane-limited and passive, i.e., $n = -h(C_I - C_{II}/R)$. Also, assume the molecule is degraded in the inner compartment with first-order rate constant k_d.
   c. Identify input, output and state variables and parameters for the most general model (i.e. the whole system). Is your system under-, over-, or exactly determined?

2. Consider the following input function:

$$u(t) = \begin{cases} 0 & t < 0 \\ 3t & 0 \le t < 3 \\ 9 & t \ge 3 \end{cases}$$

   a. Sketch the function.
   b. Find $u(s)$, its Laplace function.

3. [SEM04 #3.8] Find the $x(t)$ which solves the following integro-differential equation.

$$\ddot{x} + 3\dot{x} + 2x - 2\int_0^t e^{-\tau} d\tau = 0$$

4. [SEM #3.6] Using partial fraction expansion where required, find $x(t)$ for:

   a. $X(s) = \dfrac{s(s+1)}{(s+2)(s+3)(s+4)}$
   b. $X(s) = \dfrac{s+1}{(s+2)(s+3)(s^2+4)}$
   c. $X(s) = \dfrac{s+4}{(s+1)^2}$
   d. $X(s) = \dfrac{1}{s^2+s+1}$

5. Bequette: Chap. 3 #7.
   Also, add part (v): Solve for $y(t)$ numerically using Matlab.
   Plot your analytic and numerical solutions on the same axis.